Georgia Performance Standards Framework for MATHEMATICS – GRADE 3

Unit 2 Organizer: "MULTIPLICATION AND DIVISION OF WHOLE NUMBERS" (5 weeks)

OVERVIEW:

In this unit, students begin to understand the concepts of multiplication and division as well as learn to memorize the basic facts of multiplication and their related division sentences. This understanding of and ability to use multiplication and division is the basis for all further mathematics work and its importance cannot be overemphasized. As students move through upper elementary grades and middle school, the foundation built here will empower them to work with fractions, decimals, and percents.

To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement, but not completely replace, the textbook. Textbooks not only provide much needed content information, but excellent learning activities as well. The tasks in these units illustrate the types of learning activities that should be utilized from a variety of sources.

ENDURING UNDERSTANDINGS:

- Multiplication can be thought of as repeated addition.
- Division can be thought of as repeated subtraction.
- Multiplication and division are inverses; they undo each other.
- Multiplication and division can be modeled with arrays.
- Multiplication facts can be memorized and they can also be deduced from patterns.
- Multiplication is commutative, but division is not.
- The associative property of multiplication can be used to simplify computation.
- Square numbers and the square arrays they form result from multiplying a number by itself.

- The distributive property of multiplication over addition allows us to find partial products and then find their sum.
- Multiplying a number by ten or a multiple of ten will change the place value of the original factor's digits.
- As the divisor increases, the quotient decreases; as the divisor decreases, the quotient increases.

ESSENTIAL QUESTIONS:

- How are multiplication and addition alike and different?
- How can we picture multiplication? How can we picture division?
- What are strategies for learning multiplication facts?
- How does the associative property of multiplication help me compute?
- What is a square number?
- How are division and multiplication related?
- What does "remainder" mean in various situations?
- What happens when any number is multiplied by one? By zero? By ten or a multiple of ten?
- What is the relationship between the divisor and the quotient?

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS:

M3N3. Students will further develop their understanding of multiplication of whole numbers and develop the ability to apply it in problem solving.

a. Describe the relationship between addition and multiplication, i.e., multiplication is defined as repeated addition.
b. Know the multiplication facts with understanding and fluency to 10 x 10.
c. Use arrays and area models to develop understanding of the distributive property and to determine partial products for multiplication of 2- or 3-digit numbers by a 1- digit number.

d. Understand the effect on the product when multiplying by multiples of 10.
e. Apply the identity, commutative and associative properties of multiplication and verify the results.
f. Use mental math and estimation strategies to multiply.
g. Solve problems requiring multiplication.

M3N4. Students will understand the meaning of division and develop the ability to apply it in problem solving.

a. Understand the relationship between division and multiplication and between division and subtraction.
b. Recognize that division may be two situations: the first is determining how many equal parts of a given size or amount may be taken away from the whole as in repeated subtraction, and the second is determining the size of the parts when the whole is separated into a given number of equal parts as in a sharing model.
c. Recognize problem-solving situations in which division may be applied and write corresponding mathematical expressions.
d. Explain the meaning of a remainder in division in different circumstances.
e. Divide a 2 and 3-digit number by a 1-digit divisor.
f. Solve problems requiring division.

RELATED STANDARDS:

M3A1. Students will use mathematical expressions to represent relationships between quantities and interpret given expressions.

c. Use a symbol, such as □ and Δ, to represent an unknown and find the value of the unknown in a number sentence.

M3P1. Students will solve problems (using appropriate technology).

a. Build new mathematical knowledge through problem solving.
b. Solve problems that arise in mathematics and in other contexts.
c. Apply and adapt a variety of appropriate strategies to solve problems.
d. Monitor and reflect on the process of mathematical problem solving.

M3P3. Students will communicate mathematically.

a. Organize and consolidate their mathematical thinking through communication.
b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
c. Analyze and evaluate the mathematical thinking and strategies of others.
d. Use the language of mathematics to express mathematical ideas precisely.

M3P5. Students will represent mathematics in multiple ways.

a. Create and use representations to organize, record, and communicate mathematical ideas.
b. Select, apply, and translate among mathematical representations to solve problems.
c. Use representations to model and interpret physical, social, and mathematical phenomena.

CONCEPTS/SKILLS TO MAINTAIN:

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Odd and even numbers
- Skip count by twos, threes, fives, and tens
- Round to the nearest ten, hundred, and thousand
- Determine reasonableness using estimation
- Addition and subtraction as inverse operations
- Multiply one-digit numbers
- Commutative, associative, and identity properties of addition
- Basic addition facts
- Make tens in a variety of ways
- Double, double plus one more, addition tens, and double minus one to add quickly
- Basic subtraction facts

- Place value for ones, tens, hundreds, thousands and tenths
- Model numbers with base tens and with grid paper

SELECTED TERMS AND SYMBOLS:

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

Multiplication: The operation of repeated addition of the same number.
Factors: When two or more integers are multiplied, each integer is a factor of the product. "To factor" means to write the number or term as a product of its factors.
Product: A number that is the result of multiplication.
Array: The arrangement of objects in equal rows.
Division: The operation of making equal groups and finding the number in each group or the number of groups.
Quotient: A number that is the result of division.
Dividend: A number that is divided by another number.
Divisor: A number by which another number is to be divided.
Remainder: The number left over when a number cannot be divided "evenly".
Equal: Having the same value.
Commutative Property of Addition: The sum of a group of numbers is the same regardless of the order in which the numbers are arranged. Example: $5 + 2 = 2 + 5$ because $5 + 2 = 7$ and $2 + 5 = 7$.
Commutative Property of Multiplication: The product of a group of numbers is the same regardless of the order in which the numbers are arranged. Example: $8 \times 6 = 48$ and $6 \times 8 = 48$; therefore, $8 \times 6 = 6 \times 8$.
Associative Property of Addition: The sum of a set of numbers is the same no matter how the numbers are grouped. Example: $(3 + 5) + 2 = 8 + 2 = 10$, and $3 + (5 + 2) = 3 + 7 = 10$, so $(3 + 5) + 2 = 3 + (5 + 2)$.

Associative Property of Multiplication: The product of a set of numbers is the same no matter how the numbers are grouped. Example: (3x5)x2 = 15x2 = 30, and 3x(5x2) = 3x10 = 30, so (3x5)x2 = 3x(5x2).

Identity Property of Addition: A number that can be added to any second number without changing the second number. The identity for addition is 0 (zero) since adding zero to any number will give the number itself. Example: 0 + 5 = 5 + 0 = 5.

Identity Property of Multiplication: A number that can be multiplied by any second number without changing the second number. The identity for multiplication is "1," because multiplying any number by 1 will not change it. Example: 1 x 5 = 5 x 1 = 5.

EVIDENCE OF LEARNING:

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- use of mental math to multiply and divide;
- fluency with the multiplication facts up to 10 X 10;
- use of estimation to determine reasonableness of products and quotients computed;
- able to read, interpret, solve, and compose simple word problems dealing with multiplication and division;
- understand how to use inverses to verify accuracy of computation;
- able to write and solve expressions using symbols in place of numbers;
- able to represent numbers from tenths to ten thousandths place and accurately read as well as verbalize the same; and
- demonstrate understanding of the relatives sizes of digits in a number.

The following tasks represent the level of depth, rigor, and complexity expected of all third grade students. These tasks or a task of similar depth and rigor should be used to demonstrate evidence of learning.

- Twenty-four Kids, All in Rows
- Family Reunion
- Multiplying Money Machine
- A Giraffe Named Stretch
- Switching Numbers Around in Multiplication Stories
- Multiplication with Base-Ten Blocks
- The Quotient is Greater Than One
- Michael's Multiplication Chart
- Relating Multiplication & Division with Arrays
- Area Boxes
- Making Cents of Division
- Ice Cream Scoops

Culminating Activity: "Ice Cream Scoops"
Students will have to decide how many scoops of a particular flavor of ice cream can be purchased with the money given. Students must be able to read and interpret word problems and choose the appropriate operations to figure out what ice cream can be bought for the given amount of money. They will use both multiplication and division to solve these problems.

STRATEGIES FOR TEACHING AND LEARNING:

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols, and words.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunity to revise their work based on timely teacher feedback, peer feedback, and their own reflection.
- Students need to write in mathematics class to explain their thinking, talk about how they perceive topics, and justify their work.

Classroom Routines

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities such as taking attendance and lunch count, doing daily graphs, problem of day, and calendar activities at a math meeting board. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to interact with others during small group discussion, and how to access classroom technology such as computers and calculators.

Routinely allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. Make it a practice to write in math class by using a math journal, composing a response to the lesson such as - "what I learned today," and regularly writing to justify and explain solutions to problems.

The regular use of the routines is important to the development of students' number sense, flexibility and fluency, which will support students' performances on the tasks in this unit. See unit 1 for suggestions concerning specific ideas for classroom routines.

TASKS:

The collection of the following tasks represents the level of depth, rigor and complexity expected of all third grade students to demonstrate evidence of learning.

- **Twenty-four Kids All in Rows**

Twenty-four Kids All in Rows

Your teacher has asked you to figure out the best way to place 24 chairs in your room for a special presentation. There is only one requirement; for all arrangements, all the chairs must be placed in an array.
Show all the ways you can find to arrange the chairs. Label the arrays and write the matching number sentences.
Which of the arrangements would work well in your classroom so that all students could see the presentation? Explain why.

- Arrange 24 chairs into an array.
- Show all the ways you can find to arrange the chairs.
- Label the arrays and write matching number sentences for each example.
- Choose your favorite arrangement and explain in your math journal why that would be the best arrangement so that every student could see the presentation.

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Discussion, Suggestions, Possible Solutions

You could provide grid paper or have students draw the arrays on plain copy paper. Students should come up with a 1 x 24 array, 2 x 12 array, 3 x 8 array, 4 x 6 array, 24 x 1 array, 12 x 2 array, 8 x 3 array, and 6 x 4 array. Some students may not see the point in coming up with both the 3 x 8 array and the 8 x 3 array, for instance, but there is a definite difference when arranging chairs when you think about space in a particular room. While 8 and 3 are the factors in both arrays, both arrays have the same area, and both arrays are congruent, you would be able to see better in the third row of a 3 x 8 array than you probably would in the eighth row of an 8 x 3 array.

Twenty-four is not the only feasible number for this task. You may want to try twelve, eighteen, twenty, or even thirty or thirty-six.

Extension:

Using twenty-four, or another appropriate number, have students multiply to find the number of chairs needed for 2, 3, 4, 5, or 6 third grade classrooms that all use twenty-four chairs. They could use repeated addition, standard algorithm multiplication of a one-digit by two-digit number, or use the distributive property to show how they arrived at their solution. Is there another way to solve the problems?

For example: 24 x 3

$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \end{array}$$

or

$$\begin{array}{r} 24 \\ 24 \\ +\ 24 \\ \hline 72 \end{array}$$

or 24 = 20 + 4 so, 20 x 3 = 60 and 4 x 3 = 12 and 60 + 12 = 72.

A child could say, "I know 3 quarters equal 75 cents. So I said 75, then take away 3 because 24 is one less than 25. 75 – 3 = 72."

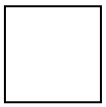
Another may say, "I counted by twenties: 20, 40, 60 – then I added 4 x 3 = 12 to get 72."

Sharing strategies is a powerful way to practice the use of the language of mathematics and for students to learn from their peers.

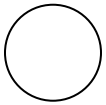
- **Family Reunion**

Family Reunion

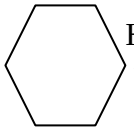
Help set up tables for your upcoming family reunion. 36 relatives need a place at a table to sit and enjoy their food and drinks. You may use the following tables.



Square tables that seat one person to an edge for a total of four people at a square table.

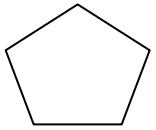


Circular bistro tables that seat exactly three people.



Hexagonal tables that seat one person to an edge for a total of 6 people.

Long, rectangular tables that seat twelve people.



Pentagonal tables that seat one person to an edge for a total of five people.

- Which table would you need the most of? Show how you would figure out how many of those tables you would need.
- Which table would you need the least of? Show how you know.
- Choose two types of tables and draw your method for seating all 36 relatives for the barbecue. Write a number sentence to describe what you've drawn.
- Suppose the only tables you had were pentagonal ones that only seat five people per table. Explain how you would seat all of your relatives.

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Discussion, Suggestions, Possible Solutions

Emphasize the connection between multiplication and division in these tasks. You may want to use a different number or even allow children to make up additional tables (octagons, rhombuses, triangles, or trapezoids) and use pattern blocks to illustrate the story. Rather than two types of tables, let students use three types that still yield seating for 36 people.
If using pentagonal tables, having a remainder is not feasible. Rather than using 7 pentagonal tables, you must use eight so that all can be seated.

Extension:

Use square tables that seat one person to an edge, but this time push the tables together end to end and see how many relatives can be seated. Continue adding tables this same way until you have enough tables to place everyone. Enter your information in a table and describe the pattern you see. How many square tables pushed end to end would it take? Choose another pattern block shape and see if the same pattern holds as you push the tables together.

# tables	# people seated	sketch	Number pattern
1	4	□	1 x 4 = 4 or (1 x 2) + 2 = 4
2	6	□□	(2 x 2) + 2 = 6
3	8	□□□	(3 x 2) + 2 = 8
4	10	□□□□	(4 x 2) + 2 = 10
5	12		(5 x 2) + 2 = 12
Etc.	Etc	Etc.	Etc.
17	36		(17 x 2) + 2 = 36

Start with a smaller number like 12 or 20. Have students predict whether the triangle pattern block will follow the same pattern and why or why not? What about the trapezoid? Hexagon?

- **The Multiplying Money Machine**

The Multiplying Money Machine

One sunny day Lucky Luke saw something shiny in the bushes. He discovered a gold box with a curious slot on the top and a hole on the side. There were three push-buttons on the front: one was red, one was blue, and one was yellow.

When he pushed the red button it said, "I double it when, you put money in." Wow! Double your money!

He pushed the blue button and it said, "I triple it when, you put money in." Triple!

Wondering what would happen next, he pushed the yellow button. It said, "Put it in, I multiply by ten."

Luke had some change in his pocket so he started trying it out. Fill in the chart below to show what happened with each coin and each button. Show all of your work.

The coins Luke had in his pocket:	Show the amount of money he got out when he pushed each colored button.			
	Red Button Double: x 2	Blue Button Triple: x 3	Yellow Button x 10	Total amount he got from the 3 coins (of the same type) that he put in the gold box
1 Nickel	+	+	=	
1 Dime				
1 Quarter				
1 Silver Dollar				
Your combination story:				

How much money did Luke start with?

_____

How much did he have at the end?

_____

How much did he profit from the gold box?

_____

Make up your own Lucky Luke story where he puts in more than one coin as well as more than one type of coin at a time. (Ex. 2 quarters and 3 dimes)

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Discussion, Suggestions, Possible Solutions

A book like Two of Everything by Lily Toy Hong would be great to read here. It's a Chinese folktale about doubling. Another suggestion is One Potato, Two Potato by Cynthia DeFelice.

Extension:
Students could use bills instead - $1, $5, $10, $20, $100. Practice using the distributive property by purposely using two of any type of coin or bill. Practice multiplying one digit by 3 digits by using dollars and cents; double $1.37 or triple $6.94.

Website:
***www.AAAMath.com** has lessons and drill & practice games addressing these same standards.*

- **A Giraffe Named Stretch**

A Giraffe Named Stretch

Write and solve multiplication stories about Stretch and his children - Bob, Sue, Amy, and the twins Ned and Fred. Here are some facts about the tallest land animal on earth.

- Giraffes have the same number legs that most dogs do -4.
- A giraffe's tongue is about 20 inches long and it is black.
- Giraffes have very long necks – 8 feet.
- A giraffe's foot is 12 inches across. That is the same size as most dinner plates!

An example multiplication story: If the tails of all the giraffes listed above were placed end to end, how long would that be? One giraffe tail is 8 feet long. There are 6 giraffes. 6 x 8 = 48 feet long altogether.

Write three more multiplication stories about giraffes or another interesting animal you know about.

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Discussion, Suggestions, Possible Solutions

Animals are usually highly motivating subjects for third graders to study. Be sure they note how science and mathematics are connected as they study life cycles throughout this school year.

Extension:
Discuss and make a list of the ways that measurements are used in science. Construct a chart to show the English measures of length and width, weight, time, speed, volume/capacity, and temperature.

Website:
http://kids.yahoo.com/animals

- **Switching Numbers Around in Multiplication Stories**

Switching Numbers Around in Multiplication Stories

13 X 7

Write a story problem where 13 is the number of groups and 7 is the number of items in each group. Show how you found the total number of items.

Write a story problem where 13 is the number of items in each group and 7 is the number of groups. Use words to explain how you found the total number of items in this story.

Write about how these two stories are connected. Explain why the stories are different but the products are the same.

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Discussion, Suggestions, Possible Solutions

Give students an example with smaller numbers and illustrate or have them act them out. Point out the connection with division. For example: If three is the number of students and five is the number of dollars each student has, the product of 3 x 5 is $15.00. If five is the number of students and three is the number of dollars each student has, the product of 5 x 3 is still $15.00. However, I would rather be a student in the first example and have $5.00 because there are fewer students.

This relates to division. If $15.00 is divided among 5 students, each one gets $3.00. However, if $15.00 is divided among only 3 students, each one gets $5.00. If the dividend remains the same, the larger the divisor, the smaller the quotient and the smaller the divisor, the larger the quotient. Third grade students need lots of examples and concrete experiences to understand these relationships.

Extension:
Read a book like: The Doorbell Rang by Pat Hutchins.
Have students write similar stories using 3- digit dividends and/ or a 3-digit factor.

Website:
http://www.lessonplanspage.com/MathLAMultiplicationDivisionUsingTheDoorbellRang23.htm

- **Multiplication with Base Ten Blocks**

Multiplication with Base Ten Blocks

Model each expression with a drawing of base 10 blocks. Show how you use the model to find the product.

2 * 13

11 * 4

5(15)

(13)(6)

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Discussion, Suggestions, Possible Solutions

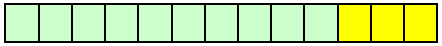
Students need to know more than one way to denote multiplication. The x becomes confusing when they start using variables so they should also recognize the raised dot and parentheses. Students would need plenty of experience with arrays and base ten blocks to be successful with this task. Some students may be able to use base ten stamps or pictures of base ten models to represent multiplication of two digit numbers. Try some students with 3-digit numbers by a 1-digit number. For example 108 x 2, 118 x 3, 129 x 4.

Detailed solutions follow below. Two colors are used to emphasize the placement of the base ten blocks.

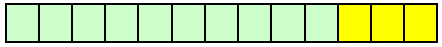
Teacher Notes:

2 x 13 *is two rows of thirteen. Have students make one row of 13 with a rod of ten and 3 units pushed together.*

10 *+ 3* = *13*

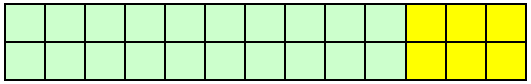


Repeat.



Place the two rows of thirteen into an array as shown below.

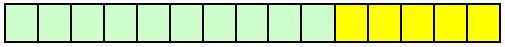
2 x 13 *2 rods pushed together make 2 rows of ten*



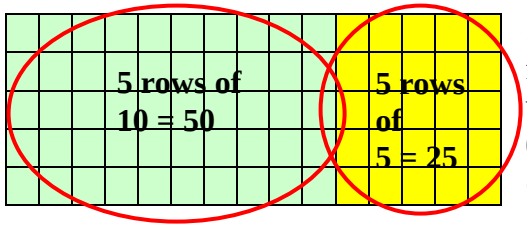
6 units pushed together to form 2 rows of 3

Students can readily see how to group the two rods to make twenty and the two rows of three units to make 6, totaling 26.

5(15) *is five rows of fifteen. Have the students make one row of fifteen with a rod of ten and 5 units pushed together.*



Repeat four more times until you have five rows of 15. Push them together to form an array of five times fifteen.



Some students will quickly catch on to the idea of multiplying the tens first, because they are so easy, to get 5 x 10 = 50. Then they will see the units as an array of 5 x 5 = 25. By using the benchmark of quarters (adding twenty-fives) they know that 50 + 25 = 75.

50 + 25 = 75

As students practice while you model on the overhead, they often become adept with this method quickly. After sufficient practice with actual base ten blocks, you might have them draw the solutions. Some will begin to do partial calculations in their heads and add them to get the totals much more quickly than they would with the traditional algorithm. This pushing together of arrays or distributive property of multiplication over addition is helpful to many children who would struggle with using only the traditional method.

The algorithm, called the partial-product algorithm, applies to algebra also.

$$\begin{array}{r} 15 \\ \times\ 5 \\ \hline 75 \end{array}$$ *can be written several ways; such as*

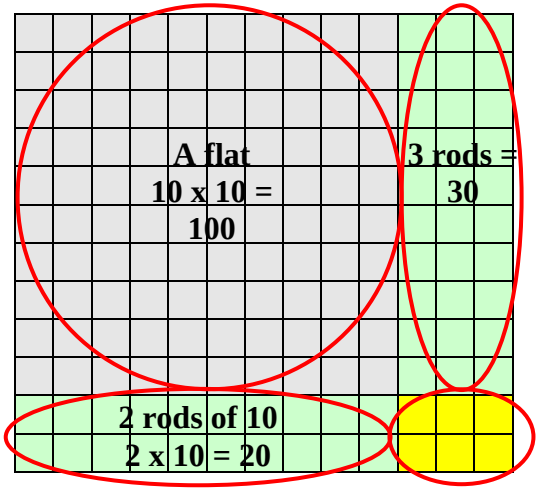
$$\begin{array}{r} 15 = 10 + 5 \\ \times\ 5 \quad \times\ 5 + \times\ 5 \\ \hline 50 + 25 = 75 \end{array}$$

$$\begin{array}{r} 15 \\ \times\ 5 \\ \hline 25 \\ +\ 50 \\ \hline 75 \end{array}$$

multiply the ones
multiply 5 x 10
add both partial products to get the total

An example of two-digit by two-digit multiplication with base tens:

12 x 13



Add all the partial products
100 + 30 + 20 + 6 = 156

Two-digit by two-digit multiplication is not required until 4th grade but you might have some students ready to explore it informally using base ten blocks.

Extension:

Give students a base ten block array or picture of the array and have them come up with the product and its factors. Allow students to decide on a number, build it with base ten blocks, then trade seats with a neighbor to find the product and its factors.

Website:
http://nlvm.usu.edu/en/nav/vlibrary.html
Try the base ten activities on the site of the National Library of Virtual Manipulatives.

- **The Quotient is Greater Than One**

The Quotient is Greater Than One

$$\frac{\square}{\triangle} > 1$$

(1) The dividend and divisor are 12 and 4. Which number goes where and why?

(2) If the dividend is 10, what is the largest number the divisor can be? Why is that true?

(3) If the divisor is 6, what is the smallest number the dividend can be? Tell why.

(4) Fill in the chart so that the quotient would be > 1. Make a new chart and fill it in so that the quotient would be less than one.

Dividend							
Divisor							

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Discussion, Suggestions, Possible Solutions

This activity requires that students prove their answers to be true. Just a simple numerical answer is not enough. Help students articulate their thinking with correct math vocabulary and make sure they understand the connection between division and fractions. Students should be able to write any division problem as a fraction and for any division problem that has a remainder they should be able to write the remainder as a fraction also. If you have students who need additional steps before they can do this work you might try these ideas: (1) They could make a Hidden Fraction Puzzle where they fill a page with fractions greater than one and put in one fraction that is less than one. Trade papers with a partner and see if you can find each other's hidden fraction. (2) They could do a Venn activity where they write fractions on index cards and create Venn Diagrams out of yarn, then have another student sort the cards into piles of greater than one and less than one.

Extension:

Have students fill in the numerator and denominator to make the quotient less than two, greater than three, equal to five, etc.

- **Michael's Multiplication Chart**

Michael's Multiplication Chart

- Michael filled in this chart to practice his multiplication facts. Which facts does he seem to know the best?
- Michael has all of his nines correct even though he has not memorized all the nine facts yet. Explain one strategy he might have used to fill in his nines on the chart.
- Michael is missing some threes and fours. How would you teach him to fill in those missing numbers? Please fill them in for him.
- How could Michael use the 4 facts to help him fill in the 8 facts? Please fill those in for him.
- Michael has done a great job filling in all the numbers on the diagonal. What kind of numbers are they?
- Draw and explain why 16 is included.
- Do you see any other patterns on the multiplication chart? Describe at least one.
- Explain how the commutative property helps you fill in facts on the multiplication chart and give an example.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18			27	30
4	8	12	16	20	24			36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36			54	60
7	14		28	35		49		63	70
8	16			40			64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

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Discussion, Suggestions, Possible Solutions

This task would work well as a math conference interview. Consider using it as an assessment during the year, adding, deleting, or changing questions as well as parts of the chart to uncover students' thinking and learning.

Extension:

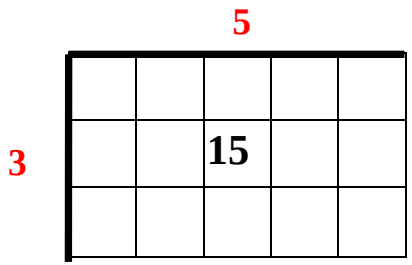
Have students fill in a multiplication chart and purposely put six wrong items. Trade with a partner and try to be the first to identify the incorrect numbers on the chart.

Website:

http://nlvm.usu.edu/en/nav/vlibrary.html
The National Library of Virtual Manipulatives

- **Relating Multiplication and Division with Arrays**

Relating Multiplication and Division with Arrays



This 3 by 5 array has a total of 15 square units.
3 X 5 = 15.

3 and 5 are **factors**.
15 is the **product**.

What is the related division sentence?
Fifteen divided by three equals five.
15 ÷ 3 = 5
15 is the **Dividend**
3 is the **Divisor**
5 is the **Quotient**

Draw the following arrays. Label the dimensions and total units. Write a multiplication and related division sentence for each one. Indicate the dividend, divisor and quotient on each one.

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Discussion, Suggestions, Possible Solutions:

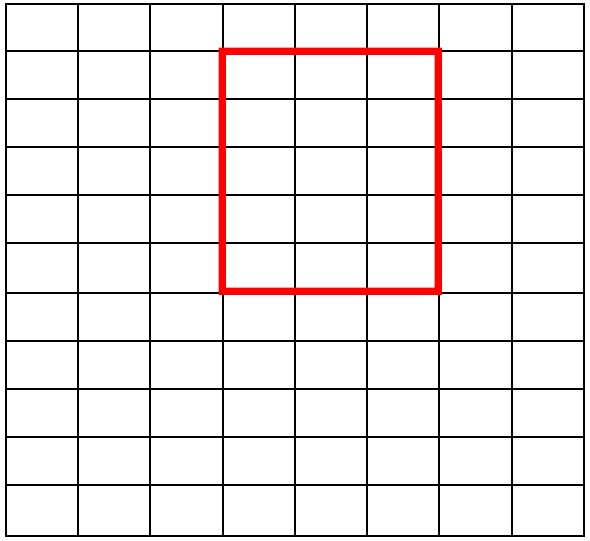
Provide grid paper for students. Be sure they understand the required vocabulary and that they can "see" the division as well as the multiplication in an array. Note the dark lines above that are intended to point out the connection between multiplication and division in the same array model.

Extension:

Have students make side-by-side models of multiplication and division with grid paper. Cut out, for instance, an array of 3 by 6 and glue it to manila paper, adding labels. Cut out another 3 by 6 array and cut it into 3separate rows of 6 and glue it beside the first array. Label all parts: divisor, dividend, and quotient to show how the two operations relate. Write a number story for each of the models.

- **Area Boxes**

Area Boxes



You will need grid paper, 2 different colors of crayons and a single die. Take turns with a partner. Roll the die twice on your turn. Your first roll is a dimension of a box. Roll the second time to get the second dimension and complete the box. Now your partner will do the same. Whoever has the box with the larger area is the winner of that round. Play ten rounds to find the overall winner.

For example:

The 1st player rolls 5 and marks a length of 5 units with his crayon.
The second roll of the die is 3. Complete the box and write the total area of the box or the array.

The 2nd player does the same and marks a box anywhere it will fit on the paper.

The player with the larger area box or array puts his or her initials in both boxes and enters the total in the chart below.

Add the numbers from all ten rounds to determine the overall winner.

Student	Round 1	Round 2	Round 3	Round 4	Round 5	Round 6	Round 7	Round 8	Round 9	Round 10	Grand Total

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Discussion, Suggestions, Possible Solutions

Discuss what makes a box "bigger." The bigger box is not necessarily the taller once since, like any array, the box could be rotated. It is bigger because the area is greater.

Extension:

A possible extension is to also look at perimeter to learn that boxes with the same area do not necessarily have the same perimeter. Have students draw a box of area 12. Ask if they can think of other boxes/arrays that would also have an area of twelve and to continue drawing until they think they have every box. It is not necessary to draw a 1 x 12 box and a 12 x 1 box since both boxes will yield the same area and perimeter. Students need only to draw boxes of non-congruent shapes. Have students find the perimeter of each box and note which shapes produce the greatest perimeter and which produce the least perimeter. Have them continue with boxes of 16, 20, 24, 30, 36 etc. You might also draw boxes of 1, 4, 9, 16, 25, etc. to discuss the fact that square numbers literally form square arrays. Square arrays have the least perimeter.

- **Making "Cents" of Division**

Making "Cents" of Division

Several students had a car wash to raise money for a field trip. They received a $20.00 bill for the first car and a $10.00 bill for the second car.

Model how the children could divide the money fairly using your paper money. Draw your solution in the table.

Where the number is blank, you decide on a different number of students and show what would happen.

3 children	____ children	9 children	____ children

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Discussion, Suggestions, Possible Solutions

Be sure they understand that "divide evenly" means there is no remainder and avoid the misconception that it means the quotient must be an even number.

If there are nine children, each will receive $3.33 and there will still be $0.03 left over. In this case, three cents is simply left over and it cannot be divided among nine children fairly. In other cases, such as dividing $30 by 4, the $2.00 remaining can still be divided further so that the four children end up getting $7.50 each. Note the difference in dividing 30 dollars by 4 and the number 30 by 4. Four is not a factor of 30; thirty is not divisible by 4. Yet, $30.00 can be dividedly evenly among four children. Discuss why this is true. Acting it out may clarify it for more children.

Extension:
Extend this lesson by having students determine which numbers divide evenly into the number thirty (or another number you choose). After seeing which numbers are factors of thirty, have them choose other divisors that leave a remainder. Which ones can be evenly divided by using coins? Which ones cannot? What is the difference in the two types of numbers? $30.00 divided among 8 children allows each of them to get $3.75 which is all the same amount with no money left over. If there were 11 children, each would get $2.72 and there would be 8 cents left over.

What if there were 30 balloons instead of $30? Or 30 students to be divided into 4 teams? Or, as in the Family Reunion task, there are 7 pentagonal tables that each seat 5, but you have 36 relatives to seat? You cannot split up a relative, or student, or balloon. What would happen in each case?

30 is divisible by	***$30.00 is divisible by***	***$30.00 divided among***	***# of children***	***leftover money***
			7	***$0.04***
1, 2, 3, 5, 6, 10, 15, 30	***1, 2, 3, 5, 6, 10, 15, 30***		***9***	***$0.03***
	And 4, 8, 12		***11***	***$0.08***
			13	***$0.10***
			14	***$0.04***

- **Culminating Task**
  This culminating task represents the level of depth, rigor and complexity expected of all third grade students to demonstrate evidence of learning.

Unit Task: "ICE CREAM SCOOPS"

The Super Delicious Ice Cream Shop has the very best ice cream in town. They have 76 flavors everyone loves and the prices are reasonable. There's just one catch. You may only order in double scoops, triple scoops, or double-double (that's four) scoops. The top selling ice creams are listed on the sign below. Find out what you can buy with $3.00. Don't worry about tax. Show all your work to prove your choice is correct.

Ice Cream Flavors and Prices For a single scoop	
Varoom Vanilla	$0.67
Cha-cha Chocolate	$1.33
Cheery Cherry	$1.04
Rockin' Rocky Road	$1.12
Stripled Strawberry	$0.89
Kid's Delight	$0.98

1. With your partner figure out the following questions: How do I find the price of a double scoop? Triple scoop? Double-double scoop?

2. What operations will I need to use to solve each of the problems? What signals me to multiply? What signals me to divide?

- Should you buy triple Varoom Vanilla or triple Cheery Cherry?
- To spend the most of your money, should you buy a double, triple, or double-double scoop of Rockin' Rocky Road? Show why. How much money would you have left?
- Which ice cream do you have enough money to pay for if you order a double-double scoop?
- On a different day, you and 5 of your friends are able to scrape together $11.76. You all want to order the same ice cream in a double scoop. Which flavor do you all buy?

Suggestions for Classroom Use

While this task may serve as a summative assessment, it also may be used for teaching and learning. It is important that all elements of the task be addressed throughout the learning process so that students understand what is expected of them.

- Peer Review
- Display for parent night
- Place in portfolio
- Photographs

Discussion, Suggestions and Possible Solutions

Begin by discussing prices of a single scoop and asking questions like: What would the price be if you ordered a double-scoop, triple scoop, or double-double scoop? What operation(s) will I need to use to find the answers? How can I use inverse operations to be sure that my work is correct? Students must recognize what multiplication is indicated by doubles, triples, and quadruples.

Students with good mental math skills will notice right away that 3 scoops of Cheery Cherry will go over $3.00, leaving Varoom Vanilla as the only reasonable choice. By the same token, $3.00 is not enough to buy triple or quadruple scoops of Rockin' Rocky Road. Students who laboriously multiply all this out need more practice in mental math skills.

Extensions:

Have students make up their own flavors and prices, use different amounts of money, and write their own Ice Cream Scoops stories to share with their classmates.