

Accelerated Mathematics 1

This is the first in a sequence of mathematics courses designed to prepare students to take AB or BC Advanced Placement Calculus. It includes radical, polynomial and rational expressions; functions and their graphs; quadratic and radical equations; fundamentals of proof; properties of polygons, circles and spheres; coordinate geometry; sample statistics and curve fitting. (*Prerequisite: Successful completion of 8th Grade Mathematics.*)

Instruction and assessment should include the appropriate use of manipulatives and technology. Topics should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic methods. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

NUMBER AND OPERATIONS

Students will use the complex number system.

MA1N1. Students will represent and operate with complex numbers.

- a. Write square roots of negative numbers in imaginary form.
- b. Write complex numbers in the form $a + bi$.
- c. Add, subtract, multiply, and divide complex numbers.
- d. Simplify expressions involving complex numbers.

ALGEBRA

Students will explore functions, solve equations and operate with radical, polynomial and rational expressions.

MA1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.

- a. Represent functions using function notation.
- b. Graph the basic functions $f(x) = x^n$, where $n = 1$ to 3 , $f(x) = \sqrt{x}$, $f(x) = |x|$, and $f(x) = 1/x$.
- c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the x - and y -axes.
- d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.
- e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.
- f. Recognize sequences as functions with domains that are sets of whole numbers.
- g. Explore rates of change, comparing constant rates of change (i.e., slope) versus variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.
- h. Determine graphically and algebraically whether a function has symmetry and whether it is even, odd, or neither.
- i. Understand that any equation in x can be interpreted as the equation $f(x) = g(x)$, and interpret the solutions of the equation as the x -value(s) of the intersection point(s) of the graphs of $y = f(x)$ and $y = g(x)$.

MA1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

- a. Simplify algebraic and numeric expressions involving square root.
- b. Perform operations with square roots.
- c. Add, subtract, multiply, and divide polynomials.
- d. Add, subtract, multiply, and divide rational expressions.
- e. Factor expressions by greatest common factor, grouping, trial and error, and special products limited to the formulas below.

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)(x - y) = x^2 - y^2$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

- f. Use area and volume models for polynomial arithmetic.

MA1A3. Students will analyze quadratic functions in the forms $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 + k$.

- a. Convert between standard and vertex form.
- b. Graph quadratic functions as transformations of the function $f(x) = x^2$.
- c. Investigate and explain characteristics of quadratic functions, including domain, range, vertex, axis of symmetry, zeros, intercepts, extrema, intervals of increase and decrease, and rates of change.
- d. Explore arithmetic series and various ways of computing their sums.
- e. Explore sequences of partial sums of arithmetic series as examples of quadratic functions.

MA1A4. Students will solve quadratic equations and inequalities in one variable.

- a. Solve equations graphically using appropriate technology.
- b. Find real and complex solutions of equations by factoring, taking square roots, and applying the quadratic formula.
- c. Analyze the nature of roots using technology and using the discriminant.
- d. Solve quadratic inequalities both graphically and algebraically, and describe the solutions using linear inequalities.

MA1A5. Students will investigate step and piecewise functions, including greatest integer and absolute value functions.

- a. Write absolute value functions as piecewise functions.
- b. Investigate and explain characteristics of a variety of piecewise functions including domain, range, vertex, axis of symmetry, zeros, intercepts, extrema, points of discontinuity, intervals over which the function is constant, intervals of increase and decrease, and rates of change.
- c. Solve absolute value equations and inequalities analytically, graphically, and by using appropriate technology.

GEOMETRY

Students will explore, understand and use the formal language of reasoning and justification. Students will apply properties of polygons, circles and spheres, and determine distances and points of concurrence.

MA1G1. Students will investigate properties of geometric figures in the coordinate plane.

- a. Determine the distance between two points.
- b. Determine the distance between a point and a line.
- c. Determine the midpoint of a segment.
- d. Understand the distance formula as an application of the Pythagorean theorem.
- e. Use the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals.

MA1G2. Students will understand and use the language of mathematical argument and justification.

- a. Use conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof as appropriate.
- b. Understand and use the relationships among a statement and its converse, inverse, and contrapositive.

MA1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

- a. Determine the sum of interior and exterior angles in a polygon.
- b. Understand and use the triangle inequality, the side-angle inequality, and the exterior-angle inequality.
- c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, HL).
- d. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.
- e. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.

MA1G4. Students will understand the properties of circles.

- a. Understand and use properties of chords, tangents, and secants as an application of triangle similarity.
- b. Understand and use properties of central, inscribed, and related angles.
- c. Use the properties of circles to solve problems involving the length of an arc and the area of a sector.
- d. Justify measurements and relationships in circles using geometric and algebraic properties.

MA1G5. Students will find and compare the measures of spheres.

- a. Use and apply surface area and volume of a sphere.
- b. Determine the effect on surface area and volume of changing the radius or diameter of a sphere.

DATA ANALYSIS AND PROBABILITY

Students will use counting techniques and determine probability. Students will demonstrate understanding of data analysis by posing questions to be answered by collecting data. Students will organize, represent, investigate, interpret, and make inferences from data. Students will determine algebraic models from data.

MA1D1. Students will determine the number of outcomes related to a given event.

- a. Apply the addition and multiplication principles of counting.
- b. Calculate and use simple permutations and combinations.

MA1D2. Students will use the basic laws of probability.

- a. Find the probabilities of mutually exclusive events.
- b. Find the probabilities of dependent events.
- c. Calculate conditional probabilities.
- d. Use expected value to predict outcomes.

MA1D3. Students will relate samples to a population.

- a. Compare summary statistics (mean, median, quartiles, and interquartile range) from one sample data distribution to another sample data distribution in describing center and variability of the data distributions.
- b. Compare the averages of the summary statistics from a large number of samples to the corresponding population parameters.
- c. Understand that a random sample is used to improve the chance of selecting a representative sample.

MA1D4. Students will explore variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations).

MA1D5. Students will determine an algebraic model to quantify the association between two quantitative variables.

- a. Gather and plot data that can be modeled with linear and quadratic functions.
- b. Examine the issues of curve fitting by finding good linear fits to data using simple methods such as the median-median line and “eyeballing.”
- c. Understand and apply the processes of linear and quadratic regression for curve fitting using appropriate technology.

Terms/Symbols: function, domain, range, zero of a function, even function, odd function, radical expression, rational expression, area model for polynomial arithmetic, volume model for polynomial arithmetic, monomial, binomial, trinomial, radical conjugates, quadratic function, standard form, vertex form, quadratic formula, discriminant, root, piecewise function, step function, extrema, point of discontinuity, conjectures, inductive reasoning, deductive reasoning, counterexample, indirect proof, converse, inverse, contrapositive, kite, incenter, orthocenter, circumcenter, centroid, points of concurrence, angle bisectors, medians of triangle, altitudes of triangle, chord, tangent, secant, central angle, inscribed angle, arc, sector, permutations (nPr), combinations (nCr), mutually exclusive events, dependent events, conditional probability, expected value, quartile, interquartile range, deviation, mean absolute deviation, curve fitting, linear regression, median-median line, algebraic model, quadratic regression

Process Standards

The following process standards are essential to mastering each of the mathematics content standards. They emphasize critical dimensions of the mathematical proficiency that all students need.

MA1P1. Students will solve problems (using appropriate technology).

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solve problems.
- d. Monitor and reflect on the process of mathematical problem solving.

MA1P2. Students will reason and evaluate mathematical arguments.

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proof.

MA1P3. Students will communicate mathematically.

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

MA1P4. Students will make connections among mathematical ideas and to other disciplines.

- a. Recognize and use connections among mathematical ideas.
- b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- c. Recognize and apply mathematics in contexts outside of mathematics.

MA1P5. Students will represent mathematics in multiple ways.

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.

Reading Standard Comment

After the elementary years, students are seriously engaged in reading for learning. This process sweeps across all disciplinary domains, extending even to the area of personal learning. Students encounter a variety of informational as well as fictional texts, and they experience text in all genres and modes of discourse. In the study of various disciplines of learning (language arts, mathematics, science, social studies), students must learn through reading the communities of discourse of each of those disciplines. Each subject has its own specific vocabulary, and for students to excel in all subjects, they must learn the specific vocabulary of those subject areas *in context*.

Beginning with the middle grades years, students begin to self-select reading materials based on personal interests established through classroom learning. Students become curious about science, mathematics, history, and literature as they form contexts for those subjects related to their personal and classroom experiences. As students explore academic areas through reading, they develop favorite subjects and become confident in their verbal discourse about those subjects.

Reading across curriculum content develops both academic and personal interests in students. As students read, they develop both content and contextual vocabulary. They also build good habits for reading, researching, and learning. The Reading Across the Curriculum standard focuses on the academic and personal skills students acquire as they read in all areas of learning.

MRC. Students will enhance reading in all curriculum areas by:

- a. Reading in all curriculum areas
 - Read a minimum of 25 grade-level appropriate books per year from a variety of subject disciplines and participate in discussions related to curricular learning in all areas
 - Read both informational and fictional texts in a variety of genres and modes of discourse
 - Read technical texts related to various subject areas
- b. Discussing books
 - Discuss messages and themes from books in all subject areas.
 - Respond to a variety of texts in multiple modes of discourse.
 - Relate messages and themes from one subject area to messages and themes in another area.
 - Evaluate the merit of texts in every subject discipline.

- Examine author's purpose in writing.
- Recognize the features of disciplinary texts.
- c. Building vocabulary knowledge
 - Demonstrate an understanding of contextual vocabulary in various subjects.
 - Use content vocabulary in writing and speaking.
 - Explore understanding of new words found in subject area texts.
- d. Establishing context
 - Explore life experiences related to subject area content.
 - Discuss in both writing and speaking how certain words are subject area related.
 - Determine strategies for finding content and contextual meaning for unknown words.